WRITTEN HOMEWORK #4, DUE APRIL 30, 2010

Unless explicitly noted, you are to justify all of your responses with work and/or proofs.

(1) Exercise 2.2 from the text.
(2) Exercise 2.5ab from the text. Part c is part of the standard theory of symmetric functions taught in algebra.
(3) Exercise 2.6 from the text.
(4) Exercise 2.7 from the text.
(5) Exercise 2.8 from the text.
(6) Exercise 2.12bcfg from the text. Strictly speaking, the problem should ask you to find all *rational* points of finite order.

Exercises 2.6 - 2.8 are purely algebra questions, and mention rings and ideals frequently. Recall that a ring is a set R with an addition and multiplication which satisfy various properties: $(R, +)$ should be an abelian group, the multiplication $\cdot$ should be associative and (at least in some definitions) have an identity element. The multiplication does not need to be commutative, nor do inverses need to exist. The addition and multiplication should satisfy an appropriate distributive property. A subset S of R is a subring if it is a ring with $+, \cdot$ inherited from R; that is, $(S, +)$ should be an additive subgroup, and S should be closed under multiplication.

An ideal I is a subset of a ring R which is a subgroup under $+$, and which also satisfies the property where if $i \in I$, then $ri, ir \in I$ for all $r \in R$. A maximal ideal is a nontrivial ideal of a ring which is not properly contained in any other nontrivial ideal ($0, R$ are the two trivial ideals of a ring R). The unit group of a ring is the set of elements of R which have multiplicative inverses in R; notice that it is not a subring since it is not closed under addition. For example, if $R = \mathbb{Z}$, then the unit group consists of ± 1.

R is a unique factorization domain if every element of R can be written uniquely as a product of irreducible elements, up to units. Recall that an irreducible element of a ring R is an element r which is not a unit, such that if $r = st$, then either s or t is a unit. All of these definitions can be found in any standard algebra text.

Suggested Exercises: If you know complex analysis, 2.3 is a good problem to work out properties of the Weierstrass series we defined in class. Exercise 2.11 is also interesting; you can use part (b) without proof in Exercise 2.12. Exercise 2.10 is the same flavor as 2.12.